Abstract—Direct, physical interaction between a human and a robot has potential to provide assistance or rehabilitation which can extend the human user's physical capabilities. To be successful, the robot must be able to realize the system-level objectives without compromising the subject's safety. For applications which involve a powered robot, compliant actuators may help guarantee safety. However, the more complex dynamics of compliant actuators requires proper model selection and controller design to give satisfactory performance. Here, a series-elastic actuator integrated to an upper-limb exoskeleton is considered. To consider the effects of the elastic element, a model for the series elastic actuator with load-side dynamics is introduced, validated, then used for controller design. The performance and safety of several control architectures are analyzed, then compared experimentally.

I. INTRODUCTION

In physical human-robot interaction (pHRI), several challenges are faced which distinguish the field from traditional control such as motion control. One area of concern is safety of the human, and formalizing safety to constraints on mechatronic design requires careful consideration. Although the final design will usually involve aspects specific to the task (e.g. range of motion, fail-safes), one general approach to guarantee safety in pHRI is by making the robot compliant. This allows the subject's interactive forces to move the robot, e.g. to avoid an undesirable configuration. Compliance has been pursued in tasks with intermittent contact (e.g. collaborative manipulators)[1], [2] as well as continuous contact (e.g. exoskeletons)[3], [4].

There are several approaches to compliance - it can be considered at the controller level as impedance/admittance control [5], [6] or at the hardware level with compliant actuators[7], [1]. For either approach, the closed-loop compliance should be carefully considered, as this is what is apparent to the subject. A compliant actuator may have open-loop dynamics which are reshaped under closed-loop control. An impedance controller which does not account for the dynamics of the interactive mechanism may present unexpected dynamics to the subject.

Although compliance can help improve safety, it must co-exist with other system objectives. In many cases, the system is to track a desired trajectory, e.g. to assist in a defined task. However, in traditional motion control, compliance is usually minimized in both the drivetrain and controller design to reduce mechanical resonance and improve rejection of load-side disturbances.

This paper investigates the compliance and motion control performance of a series-elastic actuator (SEA) driven system. The SEA can introduce underdamped higher-order dynamics when integrated to a system with a large load-side inertia, so a model which includes the load-side dynamics will first be introduced and validated. Then, two controller architectures suitable for use in pHRI experiments will be presented, and analyzed in both tracking performance and closed-loop impedance. Experimental results will compare the controllers in tracking performance, and several issues in practical implementation discussed.

II. SERIES-ELASTIC ACTUATORS

The SEA, first introduced by Pratt and Williamson [8], introduces an elastic element in the drivetrain as well as measurement of the relative displacement across this element. The output of the mechanism is no longer rigidly linked to the motor side, decoupling the output from the actuator (which may have a high impedance). By introducing a dominant elasticity with a known stiffness, position measurements across this element measure the torque through the elastic element. These actuators have proved to have practical significance in applications where interaction control is critical, such as exoskeletons [9], [10], [11], soft manipulators [1], [12] and bipedal robots [13], [14].

A. Application

This work investigates the use of SEA on an upper-limb exoskeleton, as shown in Fig. 1a. This exoskeleton is to be used in a brain-machine interface study, where the subject's desired joint-space motion is inferred from their neural activity, then realized with the exoskeleton. To realize the desired motion, the design features five powered degrees of freedom, and each joint has an elastic element with encoder measurements on either side. To evaluate the controller, experimental results shown in this paper will use the first joint, seen in Fig. 1b. As the task joint speeds are low, multi-joint dynamics are not considered here and the proposed controller has been extended to the other joints in a decentralized manner.

B. Series-Elastic Actuator Model

Several models have been adopted for SEA mechanisms. One is a fixed output model [15]. This considers the output angle of the SEA as fixed when designing a torque-mode controller which takes a reference \( \tau_{sea}^d \) and drives the motor side to realize that torque through the elastic element. Though most SEA applications will involve motion...
on the output, this model allows simpler dynamics which are tractable to traditional controller design techniques.

Another SEA modeling approach is to consider the output as freely moving[8], [4]. Again, this allows simpler analysis of the system, taking the subject side joint angle as an exogenous input to the system.

In the approach proposed here, it is desired to model the output inertia (the subject-side inertia) as well as the motor inertia, as both are considerable in this hardware. For a system where the motor-side transmission is backdriveable (e.g. ballscrews, planetary gearboxes) it may be more important to model the dynamic interaction across the spring, as any torque through the spring (τsea) not only drives the output, but is reflected to the motor-side mechanism as well. For transmissions which are not very backdriveable (e.g. high lead powerscrews, harmonic drives) this reflected spring torque will be blocked by the transmission structure and not induce motion in the motor. Here, a backdriveable planetary gearbox is used, so the motor inertia is considered.

The proposed system model is shown in Fig. 2. The motor dynamics are modeled with inertia \( I_m \), and damping \( B_m \), both of which are viewed at the output of the transmission (i.e. the geared inertia, and geared damping). A torque, \( \tau_m \) drives this actuator structure, as here the motors are operated in a torque-controlled mode. The actuator is coupled with a spring of stiffness \( K_{sp} \) to the driven inertia, \( I_s \), which in this application is the exoskeleton structure.

Choosing an appropriate model for the interaction with the subject can be challenging. In an exoskeleton application, the subject will be coupled to the subject-side mechanism, and a complete model would include the subject’s dynamics. However, developing such a model is difficult. The effective inertia may vary according to small variations in mounting location and other physiological changes in the subject. Additionally, the subject is capable of applying muscular torques to their joints, which can not be generally modeled. Therefore, this model does not account for the subject’s dynamics or muscular torques, and simply considers the torque between the subject and exoskeleton, \( \tau_s \) as the exogeneous input. A shared position, \( \theta_s \) is achieved for the subject-side mechanism and the subject.

For motion control, \( \tau_m \) will be chosen to drive the output of interest, \( \theta_s \) along a desired trajectory. This choice of model allows two major investigations. The tracking performance from a desired angle \( \theta^d_s \) to \( \theta_s \) can be examined under a given controller scheme. Additionally, the transfer function from \( \tau_s \) to \( \theta_s \) can be found, to verify the closed loop compliance which is apparent to the subject.

C. Model Identification

Before using this model for analysis, the model will be verified on the physical system. For validation in the frequency domain, a sinusoidal torque is commanded to the motor, and the resulting subject joint position is measured. In implementation, the coulomb friction in the gearbox was first estimated and compensated. The range of frequencies tested was limited by the joint limits at low frequencies and the coulomb friction model accuracy at high frequencies. Two sets of frequency data was collected, the low frequencies at an amplitude of 1.2 N-m, and the high frequencies at an amplitude of 2 N-m. The resulting data, along with the fit model is shown in Fig. 4.

The parameter values which give the fit model will then be used in the following controller design and analysis.

III. CONTROL OF SERIES-ELASTIC ACTUATORS

With the model motivated and fit, several control architectures can be proposed and analyzed. As suggested in the introduction, it is desired to do motion control on the subject-side position of this system. With the plant as modeled above, the following open-loop transfer functions can be found:
Several interesting phenomena can be seen in Fig. 5. The two complex poles arise from the higher-order dynamics coupled across the spring, giving a relatively low-frequency resonance peak. The proportional position control \( (C_p^{dir} = K_p^{dir}) \) has several undesirable properties. The system has low damping for all stable values of \( K_p^{dir} \), and transitions to unstable at \( K_p^{dir} \approx 2 \) for these parameter values. In practice, this small value of a gain cannot reject unmodeled dynamics on the subject-side, and the low damping results in unsatisfactory oscillation. Adding a fixed zero (e.g. using a PD controller, with \( K_p/K_d \) fixed) does not change either of these characteristics, as seen in Fig. 6.

### A. Direct Position Control

The most direct form of position control does not account for the compliance of the actuator, and directly chooses \( \tau_m \) according to the real subject-side joint angle \( \theta_s \) and the desired joint angle \( \theta^d \) [11]. If the compensator is of the form \( C_p^{dir}(s) \):

\[
\frac{\theta_s}{\tau_m} = \frac{G_s (1 + G_m K_{sp})}{1 + G_s K_{sp} + G_m K_{sp}}, \quad (1)
\]

\[
\frac{\theta_s}{\tau_m} = \frac{G_s K_{sp} G_m}{1 + G_s K_{sp} + G_m K_{sp}}, \quad (2)
\]

where \( G_s = (I_s s^2 + B_s s)^{-1} \), and \( G_m = (I_m s^2 + B_m s)^{-1} \).

### B. Hierarchical Control

An alternative approach is to use the well-established torque-mode control for the SEA [16], [10], [12] in a hierarchical controller. The torque mode control realizes a desired \( \tau^d_{sea} \) as a displacement across the spring, where \( \tau_{sea} = K_{sp}(\theta_m - \theta_s) \). This torque control can be used as an inner loop, and an outer motion control loop can be designed which chooses desired SEA torque. A schematic of this control scheme is shown in Fig. 7.

Let position control and inner-loop torque control be implemented as follows:

\[
\frac{\theta^d_{sea}}{\tau_m} = C_p(s) \left( \theta^d - \theta_s \right) \quad (6)
\]

\[
\tau_m = C_t(s) \left( \tau^d_{sea} - K_{sp}(\theta_m - \theta_s) \right) \quad (7)
\]

This gives the transfer functions:

\[
\frac{\theta_s}{\tau_m} = \frac{G_s G_m C_t(s) K_{sp} C_p(s)}{1 + K_{sp} H(s)} \quad (8)
\]

\[
\frac{\theta^d}{\tau_m} = \frac{G_s + G_s G_m K_{sp} (1 + C_t(s))}{1 + K_{sp} H(s)} \quad (9)
\]

where \( H(s) = G_m (1 + C_t) + G_s + C_p G_s G_m C_t \).

1) Inner Loop Controller Design: Design of an inner-loop controller can be difficult, as established techniques for hierarchical design are limited. Using other techniques to damp the system, such as frequency domain design with notch filters, has (in practice) been difficult as these shaping filters can interact with the higher-order dynamics of the
plant in unexpected ways. Another popular approach to SEA control is to use the Disturbance Observer to improve the performance of the inner-loop torque mode controller[10]. Here, the large variation in driven inertia and subject in the loop safety makes this high-gain strategy potentially undesirable.

One root locus of interest is for the transfer function from $\tau_{d\text{sea}}$ to $\theta_s$. Although the motion controller (which generates a $\tau_{d\text{sea}}$ from $\theta_s$) can move these poles under closed loop control, it would be desirable to have the inner-loop controlled poles be highly damped to begin with. This inner loop transfer function, expressed in terms of the model and controllers can be found as:

$$\frac{\theta_s}{\tau_{d\text{sea}}} = \frac{G_s G_m C_t(s) K_{sp}}{1 + K_{sp} (G_m (1 + C_t) + G_s)}$$ (10)

If, to be easily realized, the inner loop compensator is a PD controller $C_t(s) = K_{tp} + K_{td} s$ the effect of $K_{tp}$ and $K_{td}$ on the poles of (10) can be found. The effect of varying the zero location (i.e. $-\frac{K_{td}}{K_{tp}}$) is most significant, and is shown in Fig. 8. For these parameter values, the complex poles meet on the real axis at $K_{td} = 72K_{tp}$, a relatively large derivative gain.

Taking an impedance control interpretation of this outer-loop PD controller, for this system it is better to leave the ratio of stiffness to damping ($K_{td}/K_{tp}$) fixed to keep reasonable damping. There is a wide range of stiffnesses which can be realized, but as stiffnesses get larger, the system loses damping as seen in Fig. 9.

### IV. Hardware Validation

The controller structure developed in Section III-B is then implemented in hardware to validate the model-based approach, and compare performance to the direct proportional
control. In practice, an important performance criteria is the reduction of oscillation, which in the case of an exoskeleton can make the subject uncomfortable and interfere with the system level goals (trajectory tracking). The controllers, as developed, face several implementation challenges. The relatively high derivative gains make the noise from encoder differentiation more significant. However, the velocity estimates are from high resolution encoders (5000 CPR), and are reading relatively low speeds (reference joint velocities are all $|\dot{\theta}_d| < .2 \text{ rad/sec}$), thus the magnitude of derivative gains found above are feasible.

First, the step response was used to compare the controllers. The direct P controller and direct PD controller were compared with the hierarchical PD controller. The results are shown in Fig. 11.

As predicted by the model, the hierarchical controller provides a better damped response. Adding derivative control to the direct controller reduces overshoot, but the oscillation is still significant.

Next, the two controllers were compared in tracking performance. Data collected from a previous brain-machine interface experiment was taken as the joint trajectory to track, as this is representative of the intended application of this system. A portion of the response is shown in Fig. 12. The hierarchical controller achieved better tracking performance, with root mean square (RMS) position error over the entire 30 second trajectory of $\text{RMS}_{\text{hier}} = .028$. The RMS error for the direct controller was $\text{RMS}_{\text{dir}} = .065$.

One major issue in practice is the effective backlash on the spring. For small displacements, the spring does not engage. Thus, the hierarchical controller often has a larger steady state error, as small commanded $\tau_d$ do not generate any real torques in the system.

V. CLOSED-LOOP ADMITTANCE:

The closed-loop impedance $\theta_s/\tau_s$ gives the position response of the system to a force input from the subject. Again, the subject should be able to affect the position of the system, such that painful configurations can be avoided. The magnitude at low frequencies should be non-zero, such that the subject can sustain a deviation from an undesirable pose as long as needed. This means integral control should not be added to the outer motion control loop, as this would not allow the subject to permanently deviate from the system’s desired position. The impedance which can give safe performance is difficult to determine a priori, as this depends on the physical capabilities of the subject and other specifics of the task. In general, an acceptable apparent stiffness will need to be tuned iteratively, by slowly ‘stiffening’ the system during use.

As seen in Fig. 13, the subject’s ability to affect position is reduced under any sort of motion control. At higher frequencies, all the responses converge. Beyond the bandwidth
of the controller, the system presents the original subject-side dynamics. However, the hierarchical control allows the subject to more easily affect the position at lower frequencies compared to the direct controller, which can be advantageous for safety.

Under hierarchical control, to adjust the impedance apparent to the subject, the gain of the outer loop position control can be adjusted. As seen in Fig. 14, as the motion control gain is increased, the exoskeleton becomes increasingly stiff, allowing less and less motion for an equivalent subject torque. This has practical significance, as it can allow easy adjustment of the system stiffness during experiments.

Fig. 14: Apparent impedance under increasing motion control gain

VI. CONCLUSION

Implicit in the design of the controllers was the avoidance of high-gain feedback control. Although disturbance observers and other advanced loop-shaping techniques may be able to provide improved performance, any sort of high-gain feedback can be dangerous in the event of a hardware malfunction or other system error.

Another consideration for real-world implementation of any compliant actuator is unmodeled dynamics or disturbances. For unmodeled dynamics on the subject side, these forces will affect the system the same as the subject interactive torques. Without any distinguishing characteristics (e.g. different frequency bands), it is difficult to separate these disturbances from the subject’s input. Especially in low-stiffness implementations, this may compromise the system performance, so careful modeling and feedforward compensation is critical.

Also important to achieving good performance is the mechanical design. This exoskeleton has subject side dynamics which are considerable, and contain some difficult to model elements. The large inertia of the subject side mechanism puts a limit on the impedance which can be achieved at higher frequencies. Effective backlash in the mounting of the elastic element, coulomb friction in the gearbox, and other nonlinearities can also have a relatively large effect under low-gain, compliant control.

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